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A Model of Synaptic Formation Between Neurons in a Network

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Abstract: **In our days, there is a need of designing a more realistic model of a dynamic complex neural system. The inspiration arise from the biological hypothesis about the functionality of certain subsystems of the nervous system. An idea, that began in some previous papers, was materialized in the construction of the so called *prion neural system*. The present article focuses on the way the synapses between neurons of a network can be modeled by designing the binding affinities between them. Of biological inspiration is also the original idea that governs this article: in a network of neurons any neuron should not be able to bind with any other neuron. This is the reason we present a model of synaptic formation between different kinds of neurons in a network. A detailed case of study is presented in the end of this paper.**

Keywords: neural networks; hybrid intelligent systems; nature inspired computing techniques; evolutionary computing.

I. Introduction

From the biological hypothesis about the functionality of certain subsystems of the nervous system arise an idea of designing a more realistic model of a dynamic complex neural system. As in our days there is a need of designing such a system, we started to develop the *prion neural system* as part of an ambitious project to build models of complex systems into a bottom up approach. The system construction started with the molecular level and continued with the synaptic level, both of them creating the upper level of a neuron. In the diagram presented in **figure 1**, there can be seen all the levels that make up the entire system: molecular, synaptic, neuron, networks, maps, systems and the central nervous system level. The prion neural dynamic system belongs to the hybrid systems category as we needed to encompass a larger class of systems within the system structure allowing for more flexibility in modeling a dynamic biological phenomena that was already introduced in [1]. Abstracting a computing model from the structure and the functioning of a living neuron as an evolutionary multi-functional computing system was not an easy task and using only one of the directions that we already know was not enough. The design of such a system drove us to a very fruitful interdisciplinary interplay. It combines neural networks, brain calculi or membrane computing under the framework of DNA computing into this new abstract model.

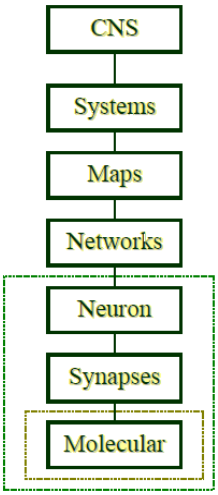

Figure. 1: The seven levels of a *prion neural system*: molecular, synaptic, neuron, networks, maps, systems and the central nervous system level.

From the computing point of view, the system works as a parallel machine at the level of its own systems, but also in a parallel manner at the level of each neuron. Regarding the neuroarchitecture and functionality, one neuron device was designed to be directly dependent on its anatomical and functional metabolism (for more information see again [1]). Each neuron itself is a system for information processing. On one hand, it offers objects (playing the role of biological protein molecules - see [1], [2] for more information) as a support for computational processes. On the other hand, it is capable of transforming the input signal into an output signal following the tradition of neural networks (read [3], [4], [5], [6] and [7] for specific details; novel neural network architectures are presented in papers like [8] and [9]). A network is designed by placing a finite set of neurons in the nodes of a finite directed graph (see [10]). All the network units of the assembly are creating a neural-like system structure modeled as a parallel distributed communication network of networks of neurons. There are two different types of interactions: local and global interactions. The local interactions are defined between neurons, meanwhile the global ones are defined between networks or neurons of different networks. All these interactions are translated as neural communication. The mean by which neural

communication can be realized is the use of a set of directed links between neurons in the entire graph. Such a directed link between two neurons is called a *synapse*. As one neuron must also communicate with the surrounding environment, we will refer to both the directed communication channels from neuron to the environment and vice-versa using the same term of a synapse. The evolution or the involution of the entire system is dependent on the synaptic creation or "deletion" from one network during the computations.

The idea that governs this article is found in the need of designing the binding affinities between neurons. This need comes from biology as not any neuron can bind to any other neuron. Creating connections depends on sufficient quantities of the corresponding substrates inside the neurons, and the compatibilities between the type of the transmitters in the pre-synaptic neurons and receptors types in the posts-synaptic neurons. The transmitters and receptors are specific "protein" objects in the pre and post-synaptic neurons respectively, in accordance with the real biological transmitters and receptors proteins (see [11], [12], [13]). To each synapse created between any two neuron devices in a network is assigned a binding affinity degree that we will refer to as the synapse weight. For the neurons with no synapse between them, the value of the binding affinity degree is assigned to zero. It is also considered to be zero the affinity degrees of the special "synapses" between neurons and the environment. The same as in real biological cases, the binding degrees (between neurons and between neurons and the environment) may suffer some changes, inducing this way both modified synaptic communications and modification into the network structure. Into a series of future articles, as it requires some special attention, we will discuss in detail both the modification that can occur into the network structure and the fact that modified synaptic communications will determine the neuron to adapt to its inputs modeling this way its behavior (as it was already described in [14]). The neuron ability to adapt to its inputs leads to a learning process at the molecular level and defines the neuron as a *feedback control system* (first time presented in [14]).

II. Background for the binding affinities modeling

In [14], we presented a naive model of synaptic formation as not any neuron can bind to any other neuron ([12], [13]). As this fact required in detail our attention, we choose to develop this idea within the pages of this article. In order to construct the background of the binding affinities modeling between neurons or between neurons and the environment, we have to remember a biological reality: the central nervous system is composed by different nervous nuclei expressing different voluntary and/or involuntary activities. Each of these nuclei is an aggregation of neurons expressing all of them the same functional activities. It is known that between neurons from the same nervous nuclei there is no directed communication, the communication being possible between neurons from different nervous nuclei. Still, we can indirectly model the communication between two neurons of the same nervous nuclei, as in biology there are no arguments that this communication can not be done ([1]). If we consider N a finite set of neurons of a network from the entire system structure, we define:
- OC_n the set of all classes of organic compounds/organic complexes for the neuron n
- $OC_{nt} \subset OC_n$ the set of all neurotransmitters and $OC_r \subset OC_n$ the set of all receptors for the neuron n
- L a finite set of labels over the English alphabet.
We define the labeling function of each organic compound/organic complex as $l : OC_n \rightarrow L$ for any n in N. As OC_{nt} and OC_r are included in OC_n we denote $T_r = l(OC_{nt}) \subseteq 2^L$ a set of labels of all neurotransmitters and $R = l(OC_r) \subseteq 2^L$ a set of labels of all receptors for the neuron n. Deriving from the biology observation that one neuron produces only one type of neurotransmitters, but it "may" have more than one type of receptors, we will consider only one label representing only one class of neurotransmitters and a number grater or equal to one of labels representing classes of receptors in n. One may say that $|T_r| = 1$ and $|R| \geq 1$ (finite).
At the molecular level, the neuron device is viewed as a system for information processing. It offers objects as a support for computational processes that transforms the input signal into an output signal (see [1] for more explanations). These objects are playing the role of biological protein molecules. We recall the *tree - like architecture* of one neuron device n defined as $\alpha(n) = \langle \alpha_{0,j_0}, \alpha_{1,j_1}, \ldots, \alpha_{r,j_r} \rangle$, where the finite number of r compartments are structured as a hierarchical tree arrangement which not only delimit protected compartments as finite spaces, but it also represent supports for chemical reactions of some chemicals embedded inside ([1] is recommended for more details). α_{0,j_0} found at the $j_0 = 0$ level of the "tree root" corresponds to the inner finite space of the neuron (containing all the other compartments), while $\alpha_{1,j_1}, \ldots, \alpha_{r,j_r}$ represent the inner hierarchical arrangement of the neuron such as for $j_1, \ldots, j_r$ natural numbers, not necessarily disjoint, with $j_k \leq r$ for $k \in \{1, \ldots, r\}$, we define the proper depth level of the k - th compartment. We have a maximum of r inner depth levels. For a better understanding of this architecture, we will consider an example.

Example 1 (A tree - like architecture.) *We will consider the neuron n. $\alpha_{0,0}$ is found at the $j_0 = 0$ level of the "tree root" and it contains all the other compartments. The inner hierarchical arrangement of the neuron architecture comprises three compartments on the first level: $\alpha_{1,1}, \alpha_{2,1}$ and $\alpha_{3,1}$. On the second level of this architecture, there are three others compartments: $\alpha_{4,2}, \alpha_{5,2}$ and $\alpha_{6,2}$. While $\alpha_{4,2}$ is the inner compartment of $\alpha_{2,1}$, the others two, $\alpha_{5,2}$ and $\alpha_{6,2}$ there are inside of $\alpha_{3,1}$. There is a last compartment found on the third level, inside $\alpha_{4,2}$. This one is $\alpha_{7,3}$. Their structure is represented in **figure 2**. In this case, the tree - like architecture of the neuron device n is defined as $\alpha(n) = \langle \alpha_{0,0}, \alpha_{1,1}, \alpha_{2,1}, \alpha_{3,1}, \alpha_{4,2}, \alpha_{5,2}, \alpha_{6,2}, \alpha_{7,3} \rangle$. The tree-like configuration arrangement of neuron n can be seen in **figure 3**.*

By the *initial configuration* we defined the initial neuronal architecture of n along with the chemicals and/or the chemical complexes found in each of its compartment regions. Formally, this was represented by

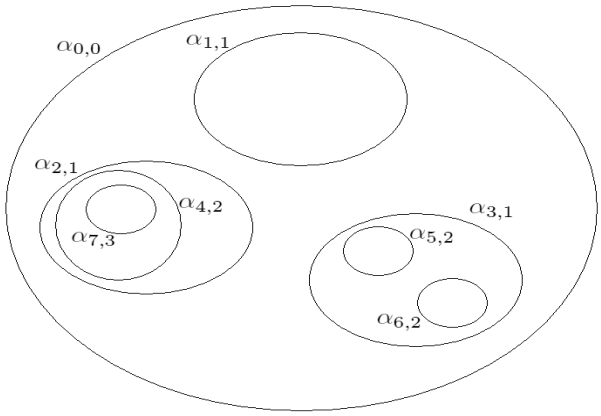

Figure. 2: An example of the neuron n having three inner depth levels and seven compartments. The inner hierarchical arrangement of the neuron architecture is defined as $\alpha(n) = \langle \alpha_{0,0}, \alpha_{1,1}, \alpha_{2,1}, \alpha_{3,1}, \alpha_{4,2}, \alpha_{5,2}, \alpha_{6,2}, \alpha_{7,3} \rangle$.

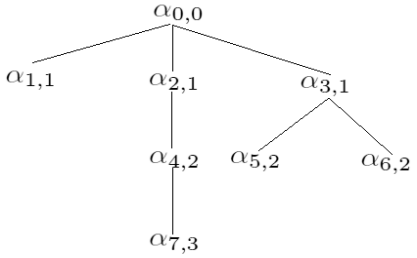

Figure. 3: The tree-like configuration arrangement of neuron n.

$(\alpha_{0,j_0} : o_0, \alpha_{1,j_1} : o_1, \ldots, \alpha_{r,j_r} : o_r)$.
For each α_{k,j_k} with $k \in \{1, ..., r\}$ we also recall the writing of o_k as the string $a_1^{m_1} a_2^{m_2} ... a_p^{m_p}$ where $p \in \mathbf{N}$, finite. Each $a_i^{m_i}$, where $a_i \in OC_n, i \in \{1, ..., p\}$ and $m_i \in \mathbf{N} \cup \{*\}$, represents the quantity m_i ($m_i \in \mathbf{N}$) of a_i found in the region α_{k,j_k} of the neuron n. If $m_i = *$, then a_i is found in an arbitrary finite number of copies in that region. We will refer to m_i as the *multiplicity* of a_i. In the representation of a configuration a compartment containing no objects inside is represented by an empty string denoted λ.

Example 2 (An initial configuration.) *Considering the tree-like architecture in the previous example, we can formally represent one possible initial configuration of the form* $(\alpha_{0,0} : a_4, \alpha_{1,1} : a_1 a_2^2 a_3^3, \alpha_{2,1} : \lambda, \alpha_{3,1} : a_1^4 a_5^3, \alpha_{4,2} : a_2^*, \alpha_{5,2} : \lambda, \alpha_{6,2} : \lambda, \alpha_{7,3} : \lambda)$.
In this initial configuration there are no objects in the compartments $\alpha_{2,1}, \alpha_{5,2}, \alpha_{6,2}$ *and* $\alpha_{7,3}$. *In* $\alpha_{0,0}$ *there is one object* a_4. *In* $\alpha_{1,1}$ *there can be found one copy of* a_1, *two copies of* a_2 *and three copies of* a_3. *Four copies of* a_1 *and three copies of* a_5 *are in* $\alpha_{3,1}$ *and there is an arbitrary number of copies of* a_2 *in* $\alpha_{4,2}$.

III. The model

It is considered the set T of discrete times defined as $T = \left\{ i \cdot \mu | i \in \mathbf{N}, \mu = \frac{1}{k}, k \in \mathbf{N}^* fixed \right\}$. For all $a_i \in OC_n$ found in the region α_{k,j_k} of n and m_i its multiplicity, we define the *quantity found into the substrate (in the region* α_{k,j_k} *of neuron* n*) of one organic compound/organic complex* a_i at the computational time $t, t \in T$ as a function $C_{n:\alpha_{k,j_k}} : T \times OC_n \to \mathbf{N} \cup \{*\}$, defined by

$$C_{n:\alpha_{k,j_k}}(t, a_i) = \begin{cases} m_i, & \text{if } m_i \text{ represents the number of copies of } a_i \text{ into the substrate} \\ *, & \text{if there is an arbitrary finite number of copies of } a_i \text{ into the substrate .} \end{cases}$$

For expressing the *quantity found into the surrounding environment (of neuron* n*) of one organic compound/organic complex* a_i at the computational time $t, t \in T$, we write the function $C_{n:e} : T \times OC_n \to \mathbf{N} \cup \{*\}$, defined by

$$C_{n:e}(t, a_i) = \begin{cases} m_i, & \text{if } m_i \text{ represents the number of copies of } a_i \text{ into the surrounding environment} \\ *, & \text{if there is an arbitrary finite number of copies of } a_i \text{ into the surrounding environment .} \end{cases}$$

Below there are presented the **properties of quantities** of organic compounds/organic complexes into the substrate.

1. The *quantity of substrate*, at the computational time $t, t \in T$ found into α_{k,j_k} is

$$C_{n:\alpha_{k,j_k}}(t) = C_{n:\alpha_{k,j_k}}(t, a_1) + C_{n:\alpha_{k,j_k}}(t, a_2) + \ldots + C_{n:\alpha_{k,j_k}}(t, a_p) = \sum_{i=1}^{p} C_{n:\alpha_{k,j_k}}(t, a_i) = \sum_{i}^{p} m_i, \tag{1}$$

for all $m_i \in \mathbf{N}$. If there is $i \in \{1, ..., p\}$ such as $C_{n:\alpha_{k,j_k}}(t, a_i) = *$, then $C_{n:\alpha_{k,j_k}}(t) = *$.

2. If at the moment t we have $C_{n:\alpha_{k,j_k}}(t, a_i) = m_i$ and at a later time $t^{'}$ a new quantity $m_i^{'}$ of a_i was produced, supposing that in the discrete time interval $[t, t^{'}]$ no object a_i was used (one may say consumed), then

$$C_{n:\alpha_{k,j_k}}\left(t^{'}, a_i\right) = C_{n:\alpha_{k,j_k}}(t, a_i) + m_i^{'} = m_i + m_i^{'}. \tag{2}$$

3. If at the moment t we have $C_{n:\alpha_{k,j_k}}(t, a_i) = m_i$ and at a later time $t^{'}$ the quantity $m_i^{'}$ of a_i was consumed, supposing that in the discrete time interval $[t, t^{'}]$ no object a_i was produced, then

$$C_{n:\alpha_{k,j_k}}\left(t^{'}, a_i\right) = C_{n:\alpha_{k,j_k}}(t, a_i) - m_i^{'} = m_i - m_i^{'}. \tag{3}$$

We make the observation that $m_i - m_i^{'} \geq 0$ $(m_i \geq m_i^{'})$ because it can not be consumed more than it exists.

4. If at the moment t we have $C_{n:\alpha_{k,j_k}}(t, a_i) = m_i$ and at a later time $t^{'}$ we have $C_{n:\alpha_{k,j_k}}\left(t^{'}, a_i\right) = m_i^{'}$, supposing that in the discrete time interval $[t, t^{'}]$ no object a_i was produced or consumed, then
   (a) if $m_i < m_i^{'}$ we say that there is a rise in the quantity of a_i;
   (b) if $m_i > m_i^{'}$ we say that there is a decrease in the quantity of a_i;

(c) if $m_i = m_i^{'}$ we say that no changes occurred in the quantity of a_i.

To model the synaptic formation we choose to work on any pair of neurons within a network structure of the system. We will design the binding affinities between any two neurons n_i, n_j in N, so we redefine
- OC_{n_i} the set of all classes of organic compounds/organic complexes for the neuron n_i and OC_{n_j} the set of all classes of organic compounds/organic complexes for the neuron n_j ($OC_{n_i} \cap OC_{n_j} \neq \emptyset$)
- $OC_{nt_i} \subset OC_{n_i}$ the set of all neurotransmitters for n_i and $OC_{r_j} \subset OC_{n_j}$ the set of all receptors for n_j. For an easier way of handling those sets we will refer to the set of neurotransmitters for n_i by OC_{nt} and the set receptors for n_j by OC_r.
- L a finite set of labels over the English alphabet.
For the labeling function of each organic compound/organic complex $l : OC_n \to L$, where n is ether n_i or n_j, we denote $T_r = l(OC_{nt}) \subseteq 2^L$ the set of labels of all neurotransmitters for n_i and $R = l(OC_r) \subseteq 2^L$ the set of labels of all receptors for n_j.
The *binding affinity* depends on a sufficient quantity of substrate and the compatibility between the type of transmitter and receptor type at time t of synaptic formation ($t \in T$).
The *sufficient quantity of substrate* is a fair ratio admitted r between the number of neurotransmitters released by n_i into the synapse and the number of receptors of the receiver neuron n_j. Without restraining generality, we consider the sufficient quantity of substrate at time t ($t \in T$) of synaptic formation as a boolean function of the assessment ratio evaluation. For $trans \in T_r$ with m its multiplicity (where for $a \in OC_{nt}$ with $l(a) = trans$ and $trans \in T_r$ we have $C_{n_i:\alpha_{0,0}}(t, a) = m$) and $rec \in R$ with n its multiplicity (where for $b \in OC_r$ with $l(b) = rec$ and $rec \in R$ we have $C_{n_j:\alpha_{0,0}}(t, b) = n$), we define $q^t : T_r \times R \to \{0, 1\}$,

$$q^t(trans, rec) = \begin{cases} 0, & \text{if } m/n \neq r \\ 1, & \text{if } m/n = r \end{cases}.$$

The function returns an answer regarding the existence of a sufficient quantity of substrate for the binding to take place, returning one if there is enough substrate to facilitate the connection and a zero otherwise.
The *compatibility function* is a subjective function $\mathcal{C}^t : T_r \times R \to \{0, 1\}$ such as for any $trans \in T_r$ and any $rec \in R$,

$$\mathcal{C}^t(trans, rec) = \begin{cases} 0, & \text{if } trans \text{ and } rec \text{ are not compatible} \\ 1, & \text{otherwise .} \end{cases}$$

This function answers the question regarding the compatibility between the neurotransmitter $trans$ in n_i and the receptor rec in n_j, as they might or they might not be compatible. The returned value is one in the case of compatibility and zero, otherwise.
The *binding affinity function* models the connection affinities between neurons by mapping an affinity degree to each possible connection. We consider $W^t \subset \mathbf{N}$ the set of all affinity degrees (at time t),

$$W^t = \left\{ w_{ij}^t | w_{ij}^t \in \mathbf{N}, \forall i, j \in \{1, 2, \ldots, |N|\}, n_i, n_j \in N \right\}.$$

For any $n_i, n_j \in N$, $trans \in T_r$ (the transmitters type of neuron n_i), $rec \in R$ (the receptors type in neuron n_j), $\mathcal{C}^t(trans, rec) = x$ with $x \in \{0, 1\}$ and $q^t(trans, rec) = y$ with $y \in \{0, 1\}$, we design the binding affinity function as a function $A_f^t : (N \times N) \times (T_r \times R) \times \mathcal{C}^t(T_r \times R) \times q^t(T_r \times R) \to W$ where
$A_f^t((n_i, n_j), (trans, rec), x, y) =$

$$= \begin{cases} 0, & \text{if } x = \mathcal{C}^t(trans, rec) = 0, \forall y \in \{0, 1\} \\ 0, & \text{if } (x = \mathcal{C}^t(trans, rec) = 1) \wedge \\ & (y = q^t(trans, rec) = 0) \\ w_{ij}^t, & \text{if } (x = \mathcal{C}^t(trans, rec) = 1) \wedge \\ & (y = q^t(trans, rec) = 1) \wedge (w_{ij}^t \neq 0). \end{cases}$$

If the neurotransmitters of one neuron and the receptors of the other neuron are not compatible, then, no matter their quantity into the substrate might be, there is no binding affinity between the two neurons. In case there is a compatibility between the neurotransmitters and the receptors of the neurons, but there is not a sufficient quantity of substrate for the binding to take place, then again the returned value of the binding affinity function is zero. It exists a binding affinity between the two neurons when there is a sufficient quantity into the substrate and $trans$ and rec are compatible. In this case, the binding affinity function models the connection affinities between neurons by mapping to it a non zero affinity degree.

Theorem 1 (The binding affinity theorem) *For $P_i \in P_{b_{n_i}}$ a finite set of biochemical processes of neuron n_i by which it produces a multiset of neurotransmitters of type $trans$ ($trans \in T_r$), at the computational time t ($t \in T$), and, in the same time, for $P_j \in P_{b_{n_j}}$ a finite set of biochemical processes of neuron n_j by which it produces a multiset of receptors of type rec ($rec \in R$), we say that there is a binding affinity between n_i and n_j with the binding affinity degree w_{ij}^t ($w_{ij}^t \neq 0$) if and only if there is $w_{ij}^t \in W^{t^*}$ such as $A_f^t((n_i, n_j), (trans, rec), 1, 1) = w_{ij}^t$.*
Proof: Immediately from the definition of the binding affinity function.

In other words the theorem says that if between two neurons there is a binding affinity and the binding affinity degree being a certain non zero value, then the returned value of the binding affinity function is precisely that non zero value. Vice-versa, if there is a non zero value representing the returned value of the binding affinity function, then between the two neurons there is a binding affinity with its degree being precisely this non zero value.

Definition 1 *For any $n_i, n_j \in N$, $trans \in T_r$ (the transmitters type of neuron n_i) and $rec \in R$ (the receptors type in neuron n_j) at time t, if there is a binding affinity between n_i and n_j with the binding degree w_{ij}^t ($w_{ij}^t \in W^t$) and $w_{ij}^t \neq 0$ then we say that **there is a connection** formed from n_i to n_j. This connection is called the **synapse between the two neurons** and it is denoted by syn_{ij}. Each synapse has a **synapse weight** $w_{ij}^t = w, w \in \mathbf{N}^*$. If instead of n_i we have e then the synapse syn_{ej} represents the directed link from the environment to the neuron n_j and if instead of n_j we have e then the synapse syn_{ie} represents the directed link*

from the neuron n_i to the environment. The synapses weighs are considered to be zero ($w_{ej}^t = w_{ie}^t = 0$).

Definition 2 *We say that there is **no connection** from neuron n_i to neuron n_j if and only if $w_{ij}^t = 0$.*

As the molecular level of a prion neural system was described in [1], along with the biological inspired phenomena that stands up as the starting idea of the system model, using the results in *Theorem 1* we will redefine in *Corollary 1* the prion neural-like network structure.

Corollary 1 *A prion neural-like network structure is a parallel distributed communication network of neurons placed in the nodes of a finite directed graph $N_s = (N, Syn)$ where N is a finite set of neurons and Syn defines a finite set of directed links called synapses, $Syn \subseteq N \times N$ a binary relation, such as $Syn = \{syn_{ij} = (n_i, n_j) | n_i, n_j \in N, i \neq j,$ with $i, j \in \{1, \ldots, card(N)\}, A_f^t((n_i, n_j), (trans, rec), 1, 1) = w_{ij}^t, w_{ij}^t \in W^{t^*}\} \cup \{(n, e), (e, n) | n \in N\}$. If instead of n_i we have e then the synapse (e, n_j) represents the directed link from the environment to the neuron n_j and $A_f^t((e, n_j), (trans, rec), 1, 1) = 0$, respectively, if instead of n_j we have e then the synapse (n_i, e) represents the directed link from the neuron n_i to the environment and $A_f^t((n_i, e), (trans, rec), 1, 1) = 0$.*
Proof: Immediately from the construction of the neural-like network structure (for more details it is recommended to be seen [1]) and the binding affinity theorem.

Underlying a conclusion, a synapse weight equals to zero if there is no connection from one neuron to the another or in the case of a direct link to or from the environment. In other words, the synapse weight w_{ij}^t equals to zero in any of the fallowing situations: if there is no connection from neuron n_i to the neuron n_j, or if instead of n_j we have e then $w_{ie}^t = 0$ (respectively, if instead of n_i we have e then $w_{ej}^t = 0$).

Theorem 2 (One way synaptic direction theorem) *For $P_i \in P_{b_{n_i}}$ a finite set of biochemical processes of neuron n_i by which it produces a multiset of neurotransmitters of type $trans$ and receptors of type rec', at the computational time t ($t \in T$), and in the same time for $P_j \in P_{b_{n_j}}$ a finite set of biochemical processes of neuron n_j by which it produces a multiset of receptors of type rec and neurotransmitters of type $trans'$, if there is w_{ij}^t in W^t with $w_{ij}^t \neq 0$ such as $A_f^t((n_i, n_j), (trans, rec), 1, 1) = w_{ij}^t$, then there is no w_{ji}^t in W^t with $w_{ji}^t \neq 0$ such as $A_f^t((n_j, n_i), (trans', rec'), 1, 1) = w_{ji}^t$. (If there is w_{ji}^t in W^t such as $A_f^t((n_j, n_i), (trans', rec'), 1, 1) = w_{ji}^t$, then $w_{ji}^t = 0$.)*
Proof: In [1] we showed that there is a restriction over the system architecture coming from a biological principle from the functionality point of view, consisting in the existence of at least one spare circuit of neurons in the network. This means that if there is a direct connection in one direction between any two neurons in the network then there can not exist the reverse direct connection between the two neurons. In our case, if $A_f^t((n_i, n_j), (trans, rec), 1, 1) = w_{ij}^t$ with $w_{ij}^t \neq 0$ for any $w_{ij}^t \in W^t$, then there can not be the reverse connection in the same time from neuron n_j to n_i. From the second definition above this means that $w_{ji}^t = 0$ leading to the conclusion that there is no $w_{ji}^t \in W^t, w_{ij}^t \neq 0$ such as $A_f^t((n_j, n_i), (trans', rec'), 1, 1) = w_{ji}^t$.

Corollary 2 *For any two neurons n_i and n_j in N, in the set of synapses $Syn - \{(n, e), (e, n) | n \in N\}$ of a prion neural system if it exists the synapse syn_{ij} from neuron n_i to neuron n_j, then the synapse syn_{ji} from neuron n_j to n_i can not exist.*
Proof: Let us consider two neurons n_i and n_j in N such as it exists the synapse $syn_{ij} \in Syn$. If this synapse exists, then it also exists a proper synapse weight $w_{ij}^t \in W^{t^*}$. From *Corollary 1* and the *Binding affinity theorem* we know that for the type $trans$ of neurotransmitters of neuron n_i and the type rec of receptors of neuron n_j we have $A_f^t((n_i, n_j), (trans, rec), 1, 1) = w_{ij}^t$, $w_{ij}^t \in W^{t^*}$. According to the *One way synaptic direction theorem* it does not exist a synaptic weight $w_{ji}^t \in W^{t^*}$ such as $A_f^t((n_j, n_i), (trans', rec'), 1, 1) = w_{ji}^t$, for any type $trans'$ of neurotransmitters of neuron n_j and any type rec' of receptors of neuron n_i (or we may say that $A_f^t((n_j, n_i), (trans', rec'), 1, 1) = 0$). *Definition 2* says that in this case ($w_{ji}^t = 0$) there is no connection from neuron n_j to neuron n_i.

Corollary 3 *For any two neurons n_i and n_j in N, if there is no connection both from n_i to n_j and from n_j to n_i at the same computational time t ($t \in T$), then there is no synapse in Syn between neurons n_i and n_j.*
Proof: It is considered $P_i \in P_{b_{n_i}}$ a finite set of biochemical processes of neuron n_i by which it produces a multiset of neurotransmitters of type $trans$ and receptors of type rec', at the computational time t ($t \in T$), and, in the same time, a finite set $P_j \in P_{b_{n_j}}$ of biochemical processes of neuron n_j by which it produces a multiset of receptors of type rec and neurotransmitters of type $trans'$. As there is no connection from n_i to n_j and there is no connection from n_j to n_i then, according to *Definition 2*, it exists the synapses weights $w_{ij}^t, w_{ji}^t \in W^t$ such as $w_{ij}^t = w_{ji}^t = 0$. But the synapse weight of a synapse between two neurons, in order for the synapse to exists (fallowing the definition of a synapse in *Corollary 1*), must have a nonzero value. In our case, this unfulfilled condition leads to the conclusion that there is no synapse in Syn between the two neurons n_i and n_j.

A. Case of study

The present case of study represents an extension to the short case of study introduced in [15]. The time $t (t \in T)$ considered is the moment of synaptic formation between neurons. Coming from the system capacity to evolve we make the observation that in time if certain conditions will be fulfilled, new synapses may be formed (or in the case of same minimum conditions not being fulfilled, some synapses can be lost). The major premises of this case of study is the existence of only one transmitter type and one receptor type for each neuron (although far distant from the biological reality). For the set $N = \{n_1, n_2, n_3\}$ of neurons, we define

- for neuron n_1: $OC_{nt_1} = \{a | C_{n_1:\alpha_{0,0}}(t, a) = m_1\} \subset$

OC_{n_1} the set of neurotransmitters type and $OC_{r_1} = \left\{x | C_{n_1:\alpha_{0,0}}(t,x) = p_1\right\} \subset OC_{n_1}$ the set of receivers type

- for neuron n_2: $OC_{nt_2} = \left\{b | C_{n_2:\alpha_{0,0}}(t,b) = m_2\right\} \subset OC_{n_2}$ the set of neurotransmitters type and $OC_{r_2} = \left\{y | C_{n_2:\alpha_{0,0}}(t,y) = p_2\right\} \subset OC_{n_2}$ the set of receivers type
- for neuron n_3: $OC_{nt_3} = \left\{c | C_{n_3:\alpha_{0,0}}(t,c) = m_3\right\} \subset OC_{n_3}$ the set of neurotransmitters type and $OC_{r_3} = \left\{z | C_{n_3:\alpha_{0,0}}(t,z) = p_3\right\} \subset OC_{n_3}$ the set of receivers type
with $m_1, m_2, m_3, p_1, p_2, p_3 \in \mathbf{N}$.

We will analyze the possibilities for synaptic formation starting from the fallowing assumptions. There are considered the mappings $l_i : OC_{n_i} \to L$ with $T_{r_i} = l_i(OC_{nt_i})$ and $R_i = l_i(OC_{r_i})$ for all $i \in \{1,2,3\}$ such as the elements $trans_i$ are labels of neurotransmitters found in T_{r_i} and the elements rec_i are labels of receptors found in R_i. There are also considered two ratios admitted, r_1 and r_2, such as $q^t(trans_2, rec_1) = 1$ (so $m_2/p_1 = r_1$) and $q^t(trans_3, rec_1) = 1$ (so $m_3/p_1 = r_2$). Four compatibilities are known: $\mathcal{C}^t(trans_2, rec_1) = 1$, $\mathcal{C}^t(trans_3, rec_1) = 1$, $\mathcal{C}^t(trans_3, rec_2) = 0$ and $\mathcal{C}^t(trans_2, rec_3) = 0$. Directly from those assumptions, we obtain the fallowing results:

- $$A_f^t((n_2, n_3), (trans_2, rec_3), 0, y) = 0 \tag{4}$$
and
$$A_f^t((n_3, n_2), (trans_3, rec_2), 0, y) = 0, \tag{5}$$
for all $y \in \{0,1\}$, leading to the impossibility of any existence of any bound between neurons n_2 and n_3, nether at time t nor at any further moment in time as long as they produce the same pairs of neurotransmitters and receivers as those produced at time t. This comes from the incompatibility of the two neurons in creating a synapse between them (see *Corollary 3*). We say that there is no syn_{ij} and no syn_{ji} for $i, j \in \{2,3\}, i \neq j$.
- there is $w_{21} \in \mathbf{N}$ such as
$$A_f^t((n_2, n_1), (trans_2, rec_1), 1, 1) = w_{21}, w_{21} \neq 0. \tag{6}$$
We say that there is a binding affinity between neurons n_2 and n_1 with the degree w_{21}.
- there is $w_{31} \in \mathbf{N}$ such as
$$A_f^t((n_3, n_1), (trans_3, rec_1), 1, 1) = w_{31}, w_{31} \neq 0. \tag{7}$$
We say that there is a binding affinity between neurons n_3 and n_1 with the degree w_{31}.

In this moment our study can lead into two different directions of interpretation considering the fact only few parameters are known. For example, we also know the incompatibility of n_2 and n_3 neurons in creating a synapse between them and the convenient binding affinities between n_2 with n_1 and n_3 with n_1. But what about the other possibilities in creating (or not) synapses (for example from n_1 to n_2)?

(A) If we consider the assumptions being mandatory in the sense those premises being the favored ones in creating synapses, then there are formed the synapses syn_{21} and syn_{31} between neurons n_2 with n_1 and n_3 with n_1, while, there is no syn_{ij} and no syn_{ji} for $i, j \in \{2,3\}, i \neq j$. More of that, as long as the neurons produce the same pairs of neurotransmitters and receivers as those produced at time t, no matter if $trans_1$ with rec_2 or rec_3 have or have not a sufficient quantity of substrate or are or are not compatible, the synapses syn_{12} and syn_{13} can not exist. So, the final set of synapses is $Syn = \{syn_{21}, syn_{31}\}$ and the neural network structure $N_s = (N, Syn)$ is formed.

(B) If we consider the assumptions not being mandatory in the sense of the input premises representing only the known conditions (one may say the implicit assumptions), we will discuss below all the possibilities of synaptic formation for this case of study:

1. For $\mathcal{C}^t(trans_1, rec_2) = 0$, we have
$$A_f^t((n_1, n_2), (trans_1, rec_2), 0, y) = 0.$$
As the binding from n_1 to n_2 is not possible, the synapse syn_{21} can be formed.
   (a) if $\mathcal{C}^t(trans_1, rec_3) = 0$, then
   $$A_f^t((n_1, n_3), (trans_1, rec_3), 0, y) = 0.$$
   As the binding from n_1 to n_3 is not possible, the synapse syn_{31} can be formed according to (7). The final set of synapses is $Syn = \{syn_{21}, syn_{31}\}$ and the formed neural network structure is the same one as the one in (A).
   (b) if $\mathcal{C}^t(trans_1, rec_3) = 1$, but $q^t(trans_1, rec_3) = 0$, then
   $$A_f^t((n_1, n_3), (trans_1, rec_3), 1, 0) = 0.$$
   As the binding from n_1 to n_3 is not possible, the synapse syn_{31} can be formed according to (7). The final result is the same one as in the case (1a).
   (c) if $\mathcal{C}^t(trans_1, rec_3) = 1$ and $q^t(trans_1, rec_3) = 1$, then
   $$A_f^t((n_1, n_3), (trans_1, rec_3), 1, 1) = w_{13},$$
   with $w_{13} \neq 0$. In the same time there is $w_{31} \neq 0$ such as
   $$A_f^t((n_3, n_1), (trans_3, rec_1), 1, 1) = w_{31}.$$
   We are facing with two possibilities (*Theorem 2*): either will be formed the synapse syn_{31} and not syn_{13}, the final result being the same one as in the case (1a), either will be formed the synapse syn_{13} and not syn_{31} and the set of synapses being $Syn = \{syn_{21}, syn_{13}\}$.

2. For $\mathcal{C}^t(trans_1, rec_2) = 1$, but $q^t(trans_1, rec_2) = 0$ we have the same returned value of the binding affinity function as in case (1):

$$A_f^t((n_1, n_2), (trans_1, rec_2), 1, 0) = 0.$$

A binding from n_1 to n_2 is not possible. In this case the synapse syn_{21} can be formed. All the further discussions in this case will be the same ones as those from (1a,b and c).

3. For $\mathcal{C}^t(trans_1, rec_2) = 1$ and $q^t(trans_1, rec_2) = 1$, we have

$$A_f^t((n_1, n_2), (trans_1, rec_2), 1, 1) = w_{12},$$

with $w_{12} \neq 0$. In the same time there is $w_{21} \neq 0$ such as

$$A_f^t((n_2, n_1), (trans_2, rec_1), 1, 1) = w_{21}.$$

We are facing again with two possibilities (*Theorem 2*):

(a) if it will be formed the synapse syn_{21} and not syn_{12}, then we are finding the same situations as those from (1a,b and c)

(b) if it will be formed the synapse syn_{12} and not syn_{21}, then the cases remaining to be discussed below are between neurons n_1 and n_3 as fallows:

i. $\mathcal{C}^t(trans_1, rec_3) = 0$, we have

$$A_f^t((n_1, n_3), (trans_1, rec_3), 0, y) = 0.$$

As the binding from n_1 to n_3 is not possible, the synapse syn_{31} can be formed. The final set of synapses is $Syn = \{syn_{12}, syn_{31}\}$

ii. $\mathcal{C}^t(trans_1, rec_3) = 1$, but $q^t(trans_1, rec_3) = 0$, we have

$$A_f^t((n_1, n_3), (trans_1, rec_3), 1, 0) = 0.$$

Again, a binding from n_1 to n_3 is not possible and the set of synapses that can be formed is the same one as in (b.1.) ($Syn = \{syn_{12}, syn_{31}\}$).

iii. $\mathcal{C}^t(trans_1, rec_3) = 1$ and $q^t(trans_1, rec_3) = 1$, we have

$$A_f^t((n_1, n_3), (trans_1, rec_3), 1, 1) = w_{13},$$

with $w_{13} \neq 0$. In the same time there is $w_{31} \neq 0$ such as

$$A_f^t((n_3, n_1), (trans_3, rec_1), 1, 1) = w_{31}.$$

We are again confronting with two possibilities (*Theorem 2*): either will be formed the synapse syn_{31} and not syn_{13}, the final synaptic formation leading to the same set of synapses as in (b.1.), either a new synapse will be formed, this one being syn_{13} and not syn_{31}. The final network configuration will be $N_s = (N, Syn)$, where $Syn = \{syn_{12}, syn_{13}\}$.

IV. Conclusion

In the present paper we modeled synaptic creation between neurons by designing the binding affinities between them. There are some obvious conclusions arise from the model. First of all, concerning *the existence* of a synapse between two neurons, we have to note its one-way directionality. Of course, in the case of synapses between neurons and the environment this is not necessarily obligatory. The second conclusion refers to a synapse between any two neurons in the network to *not exist*. In this case, there should be no connections in any sense (from one neuron to another, nor vice-versa). The last conclusion came from the fact that changes, in time, of the the binding degrees (between neurons and between neurons and the environment) will induce modified synaptic communications and modification into the network structure. Our work will continue into a series of future articles by describing the way modified synaptic communications will determine the neuron to adapt to its inputs by modeling, this way, its behavior. Due to the fact that the entire complex system structure and functionality is still in its early stages of a theoretical model, experiments to test the proposed technique will be done. The purpose of this paper was to create a stronger background than the one already introduced in [14]. This background represents the platform on which one neuron device develops as a feedback control system by its ability to adapt to its own inputs leading, this way, to a learning process at the molecular level. Precisely the implementation at a molecular level of a neuron model viewed as a feedback control system is expected to have major impact in the future of biological, medical and information processing areas.

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